JOURNAL OF URBAN ECONOMICS 7, 2–19 (1980)

Costs of Adjustment and Demolition Costs in Residential Construction, and Their Effects on Urban Growth[1]

ODED HOCHMAN

Ben Gurion University, Beersheva

AND

DAVID PINES

Tel Aviv University

Received March 1, 1977: revised November 16, 1977

Dynamic models in Urban Economics are still a new phenomenon. The difficulty in the development of such models stems from two main reasons. First is the technical difficulty in incorporating both time and space into a single model, and second is the early stages of development of urban economic theory at present.

Among the early contributions to urban economic dynamics are the papers by Fujita [3, 4]. We refer here to the recent one (1976). In this work he developed a methodological basis for dynamic modeling in urban economics. He investigated growth patterns of a city with a given growth rate. The author investigates the properties of demand and supply, bid rents and prices, and establishes their growth patterns.

The economic interpretation of the model is somewhat weak. The difficulties stem from the assumption that the demand for housing in each zone consists of a single point in the price-commodity space at a given time. This assumption, probably inherited from linear programming spatial models, beside being difficult to explain, also yields unstability in the model and prevents the author from generating an equivalent completely decentralized equilibrium model. The model investigated in

[1] This research was performed in the University of Chicago and Tel Aviv University. Partial support for this research was provided by NSF (Grant ENV 75-23397) and the Center of Urban Studies, Tel Aviv University. We are indebted to H. Ben Shahar, E. Berglas, A. Razin, E. Sheshinski, and C. Upton for detailed and useful comments on previous versions of the paper. We are grateful to R. E. Lucas for encouraging remarks and to E. S. Mills for a thorough Editorial review.

0094-1190/80/010002-18$02.00/0

this paper is similar in structure to Fujita's paper. The approach adopted by us to avoid discussing the demand side, contrary to Fujita, was to assume a completely inelastic demand curve at each point in time. Another simplifying assumption which make Fujita's model poorer than it should be, is that each type of building consumer (population and offices) demands only one type of building. It would have enriched the Fujita's results considerably if each population type could reside in several types of buildings, each with a different amount of land per building unit. This assumption is common in the static literature and it would be interesting to observe its effects through time.

In our model, although we restrict ourselves to one population type, we allow a continuum of housing densities, and since we are interested in adjustment costs we allow those densities to vary with time, at a cost. We also include in our model maintenance and depreciation costs, a very important factor when dealing in dynamic phenomena and which were not previously included in urban models.

Thus we cannot only characterize the optimal solution but also investigate fully the completely decentralized competitive market and find out under what circumstances the competitive solution is optimal.

In this work we investigate dynamic aspects concerned with urban growth. An investment process is often followed by extra costs related to the rate of investment. It is true for the economy as a whole (see, for example, [2, 18]), or in the case of a single firm [5, 6, 9, 17]. In urban development, investment or construction activity is the main aspect of development activity. Urban investment is confined in space and adjustment costs are more likely to occur than in any other activity. Additionally, investments in urban development are long term, irreversible, and specific. All this implies that current investment decisions have a strong effect on future development. Nevertheless, no attempt has been made so far to analyze the problem. The reason for this is probably the technical difficulty involved in such a theory, which may have hampered past attempts to solve the problem. The reader will find that, although simplifying assumptions are made, our model is still complex. However, we get interesting results that seem to apply to more general models as well.

Apart from adjustment costs, we assume that marginal construction and maintenance costs increase with density. Those assumptions are common in the urban economic literature. (For example, see [11]). When demolition is required for further development, demolition costs are the alternative to adjustment costs. A similar cost component is discussed by Rothschild [15]; demolition cost is discussed as well in this paper.

The optimal solution of the model is compared to the competitive equilibrium solution. Conditions for competitive equilibrium to be optimal are then derived. The main findings of the paper are:

(1) The more rapid the rate of growth of a city, the more sprawled and scattered it becomes. Therefore, we expect to find that where two cities are of equal size, the newer city will be more sprawled and scattered.

(2) In rehabilitation areas where massive demolition takes place, the optimal construction policy is to concentrate all construction activity in one zone at a time.

(3) Under rational expectations and where adjustment costs are dominant, market equilibrium yields the optimum. In zones where demolition costs dominate, a free market leads to a monopolistic solution or no solution at all, and thus government intervention is required.

(4) Cities in competitive equilibrium with unanticipated high (low) rates of development tend to be dense (sprawled) with respect to their optimal size.

The plan of the paper is as follows: Section 1—the assumptions of the model are presented and interpreted. Section 2—the formal model is presented, and the necessary and sufficient conditions for the optimal solution are derived and interpreted. Section 3—the optimal solution to the case with costs of adjustment is derived. Section 4 is devoted to the optimal solution of the case with demolition costs. Section 5—the competitive equilibrium solution of the problem is derived and compared to the optimal solution. Conclusions and evaluation of the results follow in Section 6.

I. THE MODEL ASSUMPTIONS

Following traditional literature [11, 12], we assume a concentric city with residential areas centered around the CBD and surrounded by the agricultural ring. We limit attention to the residential ring assuming the CBD is given. Divide the area around the CBD into concentric rings $i (i = 1, 2, \cdots)$, each with an equal area S. Without loss of generality, assume $S = 1$. Zone i is represented by a single parameter $T_i(t)$ which represents commuting costs to the CBD per household per unit of time at time t. We further assume that $T_i(t)$ does not vary with time, i.e., $T_i(t) = T_i$. This is a strong assumption since in recent literature transportation costs are assumed to depend on congestion which in turn depends on the distribution of the population [7, 10, 16]. However, we will show that our main results are valid in the general case as well. It is also assumed that $T_i \to \infty$ as $i \to \infty$. Let $H_i(t)$ be the number of households located in ring i at period t. Since $S = 1$, $H_i(t)$ is also population density in zone i. The number of households located within the city at period t is predetermined and equal to $n(t)$. We assume $n(\infty) < \infty$ and $dn(t)/dt > 0$. Hence:

$$\sum_{i=1}^{\infty} h_i(t) \geq \dot{n}(t), \tag{1}$$

where

$$h_i(t) = \dot{H}_i(t) \tag{2}$$

and a dot denotes derivation with respect to time. We omit the index t when possible.

It is assumed that the quantity of housing supplied to each household is constant. Without loss of generality, assume this quantity is unitary.[2] Since attention is focused on production aspects of the problem, this assumption is convenient and not too restrictive. For similar and stronger assumptions under similar circumstances [see, 10, 11].

The cost of producing $h_i(t)dt$ units of new housing and maintaining $H_i(t)$, the existing stock, is $g[H_i(t), h_i(t)]dt$. $g[H_i(t), 0]dt > 0$ is the cost of maintenance of existing stock H_i and $g[0, h_i(t)]dt > 0$ is the cost of building $h_i(t)dt$ new units of housing when there is no existing stock. We assume the following properties for the function $g(H, h)$:

$$g(0, 0) = 0. \tag{4.1}$$

$\partial g(H, h)/dh = g_h > 0$ is the marginal cost of a new constructed unit of housing given a stock H. (4.2)

$\partial g(H, h)/dH = g_H > 0$ is the marginal cost of maintaining a unit of stock, while new construction is taking place at the rate of h. (4.3)

$$\partial^2 g(H, h)/\partial h \partial H \equiv g_{hH}(H, h) > 0. \tag{4.4}$$

This assumption means that the marginal cost of new construction increases with the existing stock. That it costs more to add a unit to high density than to low density areas, which takes account of the fact that a unit in a high rise building costs more, in terms of production factors other than land, than a unit in low buildings. The disturbance factor in terms of pollution, noise and congestion to the existing population also contributes to this factor.

$$\partial^2 g(H, h)/\partial H^2 = g_{HH}(H, h) > 0 \tag{4.5}$$

[2] Housing units can differ, in different locations, in size, height of building, land-capital ratio, etc. However, they all have to supply the same services—i.e., consumers are indifferent between them.

is the same as the former assumption, but with regard to maintenance costs.

Notice that even if there are economies of scale in construction, or maintenance, they exist when all the production factors, including land, are taken into account. In this case, however, land is held constant and only other factors (labor and capital) are permitted to change (for details see [11]). Hence, it is logical to assume that no economies of scale exist in the reduced cost function.

$$\partial^2 g(H, h)/dh^2 = g_{hh} \neq 0. \tag{4.6}$$

There are two possible situations.

$$g_{hh} > 0 \tag{4.6a}$$

is the case of cost of adjustment. In this case, the greater the rate of construction in a given zone, the higher are the costs per additional unit. This is explained by the fact that the quantity of land in a zone is fixed. Hence, higher rate of construction means higher concentration of labor and machinery in the same site-size in order to hasten construction. This concentration causes bottlenecks and inefficiencies and is expensive. All this is true for fixed factor prices, but an increase in the construction rate causes factor prices to rise for a given density in the area. Consider labor as an example. Its supply curve is determined by the number of households in the area and is, hence, an upward sloping curve for a given density. The demand for labor is determined by the production rate at each instant. Given that the production rate in all other industries remains constant, a higher rate of residential construction implies an upward shift in the demand curve for labor and, hence, an increase in current equilibrium labor prices. This is the classical justification of costs of adjustment; see for example Lucas. A similar argument may also apply to other construction factors like machinery, raw materials, etc. This effect will increase when the area becomes more populated. Another cause is the disturbance factor; it is assumed that the higher the rate of construction, the higher is the disturbance cost per unit of new housing.

For small enough H, g_h is nearly constant with h, but when H increases and vacant land becomes scarce, then g_h increases with h. As long as we do not have to tear down houses in an area in order to construct new buildings, costs of adjustment are dominant. When we cannot build up a zone without demolishing existing buildings, a new factor is introduced: the cost of demolition which consists mainly of the loss of services of existing units. In this work, demolition in an area is possible only if the area will be rebuilt more densely. Demolition cost has to be assigned to the additional units added to the area. If the zone is rebuilt quickly, we can build more new dwelling units on the site evacuated by demolition than if the zone is rebuilt slowly.[3] Hence, in demolition costs the marginal construction costs reduce as the rate of construction increases. We still have the effect of adjustment costs so that even where demolition takes place, g_{hh} might be positive. It is reasonable to assume, therefore, that

$$g_{hh} < 0 \tag{4.6b}$$

in regions where most or all of the area has to be demolished and rebuilt and in zones where the cost of adjustment factor is reduced. Rehabilitation areas fit this description. We can, therefore, conclude that in most places adjustment costs are dominant except maybe in densely populated areas near the CBD, destined for rehabilitation.

II. THE OPTIMIZATION MODEL

Following Mills and de Ferranti, let the optimal construction schedule be the one that minimizes the discounted total residential costs for the given population schedule. The problem is, therefore, to minimize the total cost function,

$$C = \int_0^\infty e^{-rt}\{\sum_{i=1}^{\infty} [H_iT_i + g(H_i, h_i)]\}dt, \tag{5}$$

subject to (1), (2), and (6)

$$h_i \geq 0 \quad i = 1, 2, \ldots. \tag{6}$$

Necessary and sufficient conditions are given by equations (7) to (10), (7) and (8) being the necessary conditions,

$$\lambda - q_i - g_h(H_i, h_i) \leq 0, [q_i + g_h(H_i, h_i) - \lambda]h_i = 0, \tag{7}$$

$$\dot{q}_i = rq_i - T_i - g_H(H_i, h_i). \tag{8}$$

The transversality condition is given by (9)

$$\lim_{i\to\infty} \sum e^{-rt}q_i(t)H_i(t) = 0. \tag{9}$$

The second order conditions are given by (10)

$$(g_{HH}g_{hh} - g_{hH})/g_{hh} > 0 \text{ for } g_{hh} \neq 0, \quad g_{HH} > 0 \quad \text{for } g_{hh} = 0. \tag{10}$$

[3] Higher buildings require less land per dwelling unit than lower buildings. If we have to build a greater number of dwelling units in a built area, we can construct higher buildings and hence use less land per unit and thus save in demolition costs per unit.

For $g_{hh} < 0$, (10) is always fulfilled; for $g_{hh} > 0$ it is fulfilled once $g(H, h)$ is convex, which is assumed henceforth. The new variables are

q_i—abbreviated form for $q_i(t)$, the reciprocal of the auxiliary function of $H_i(t)$;

λ—abbreviated form of $\lambda(t)$, the Lagrange multiplier of equations (6); equations (7) to (9) imply (11)

$$\lambda - g_h - T/r - e^{rt} \int_0^{\infty} e^{-rt'} g_H(t') dt' \leq 0$$

$$\left[\lambda - g_h - T/r - e^{rt} \int_t^{\infty} e^{-rt'} g_H(t') dt' \right] h = 0. \tag{11}$$

The economic interpretation of (11) is as follows:

λ = marginal cost of the additional households for the city as a whole.
g_h = direct marginal cost of the housing stock in a given subarea i.
T_i/r = current value of the transportation costs from t to infinity in the given subarea.

$e^{rt} \int_t^{\infty} e^{-rt'} g_H(t') dt'$ = the discounted cost to time t of the marginal unit of the housing stock at time t. (Reflecting the increase in costs of maintenance of future housing as a function of an increase in the existing housing stock by one unit)

Hence, (11) implies that nowhere is the total cost of accommodating a household smaller than the marginal cost in the city. The housing stock remains constant when its marginal costs exceed the marginal cost of accommodating a household in the city as a whole.

III. THE SOLUTION FOR $g_{hh} > 0$

Assume, as a working assumption to be dropped later, that $\lambda(t) = \lambda_0$ is constant and given, and solve for the other variables including $n(t)$. Each subarea can now be dealt with independently of the others. Let us first describe the optimal trajectory of a given zone, in the qH plane. From (7) and (8) we get

$$\left.\frac{\partial q}{\partial H}\right|_{\dot{H}=0} = -g_{hH} < 0, \tag{12}$$

$$\left.\frac{\partial q}{\partial H}\right|_{\dot{q}=0} = \frac{g_{hh} g_{HH} - g_{hH}^2}{g_{hH} + r g_{hh}} > 0, \tag{13}$$

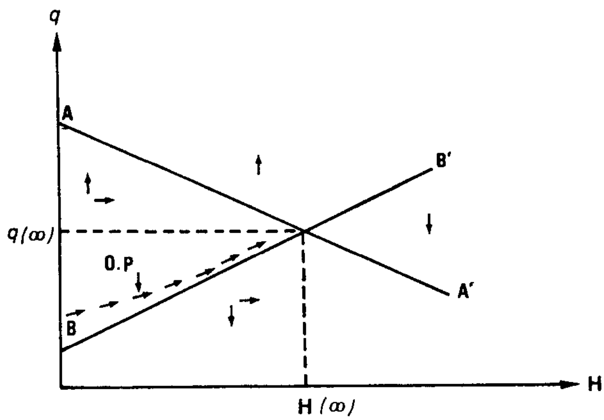

FIGURE 1

$$\left.\frac{\partial \dot{q}}{\partial q}\right|_{dH=0} = r + g_{Hh}/g_{hh} > 0, \tag{14}$$

$$\left.\frac{\partial \dot{H}}{\partial H}\right|_{dq=0} = -g_{hH}/g_{hh} < 0. \tag{15}$$

From (7) and (8) using (12) through (15) we can draw the phase diagram described in Fig. 1.

As long as Point B is below Point A in Fig. 1, a steady state, which is the intersection of AA' and BB' must exist and with it an optimal path leading to it. The condition that A is above B is

$$\lambda > g_h(0, 0) + g_H(0, 0)/r + T/r.$$

If the above inequality is not fulfilled, inequality holds in (7), and hence no development will take place in the zone.

Define the city boundary as zone $i = L$ which is the first zone not being built. $i = L$ will be the border of the city if (16) is fulfilled.

$$T_L(t) = T_L[\lambda(t)] = r\lambda - rq_h(0, 0) - g_H(0, 0). \tag{16}$$[4]

We see from (16) that for constant λ, T_L is constant and does not vary with time.

We now compare the optimal path of two zones, i_1 and i_2, where $i_1 < i_2$ and hence $T_{i_1} < T_{i_2}$. The line AA' in Fig. 2 is the same for i_1 and i_2. The line $B_2B'_2$ for i_2 intersects AA' at S_2 and the line $B_1B'_1$ for i_1

[4] A more accurate description is

$$T_L = \min\{T_i > r\lambda - rg_h(0, 0) - g_H(0, 0)\}.$$

We will use (16), however which assumes T_i is continuous, to simplify the analysis.

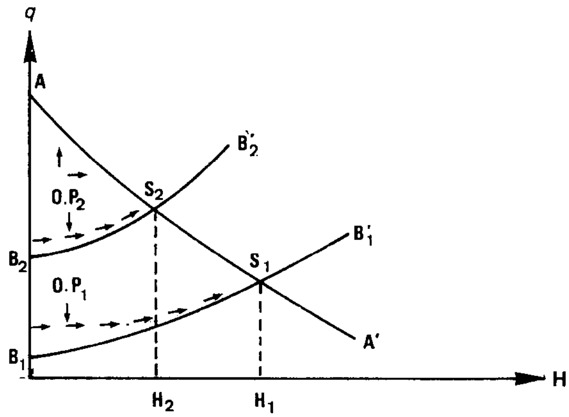

FIGURE 2

intersects AA' at s_1. From Fig. 2 it follows that $H_1 > H_2$. Therefore, the whole optimal path of zone i_2 is to the left and above the *o.p.* of zone i_1. It can be easily shown that two optimal paths of two different zones never intersect.

We can now sum up the results for a constant λ in the following propositions:

(a) The distance of the boundary of the city is finite and constant.

(b) Every subarea which is built up at the steady state is continuously developed from the start of the development process.

(c) The closer the subarea is to the center, the higher are the rate and level of development, i.e., the higher are h and H. This follows from the comparison between two different zones made above and the assertion that any two optimal paths cannot intersect. This implies that the lower the optimal path in Fig. 2 the higher the rate of development for a given H, including $H = 0$. Hence from the beginning, the subarea with lower optimal path (Fig. 2) must be more developed than the one with the higher optimal path.

(d) Constant $\lambda(t)$ implies upper bounds to the size of the population. This follows from the fact that the number of developed subareas is finite, and a steady state exists in each subarea developed.

We can now drop the assumption of constant $\lambda(t)$ and see what happens when we let λ change. We begin this analysis by examining the effects of a change in λ on the optimal path in a given zone. By differentiating (7), we get

$$(\partial q/\partial \lambda)\,|_{\dot{H}=dH=0} = 1, \tag{17}$$

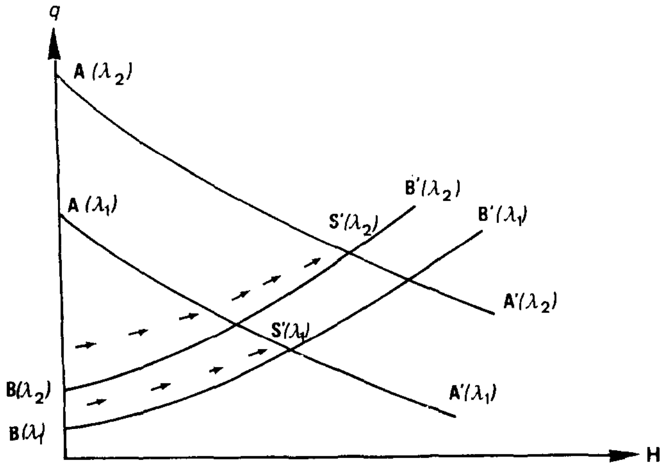

FIGURE 3

and from (7) and (8) we get

$$(dq/d\lambda)\,|_{\dot{q}=dH=0} = \frac{g_{Hh}}{g_{hh}r + g_{Hh}} > 0 \qquad \text{and} \qquad <1. \tag{18}$$

Since $(dq/d\lambda)\,|_{\dot{q}=dH=0}$ is greater than zero and less than 1, we have $0 < (dq/d\lambda)\,|_{\dot{q}=dH=0} < (dq/d\lambda)\,|_{H=dH=0} = 1$.

It follows that both AA' and BB' move upwards when λ increases, but AA' moves more than BB', so the two lines move away from each other. This is reflected in Fig. 3.

It is clear from Fig. 3 that if $\lambda_2 > \lambda_1$, $S(\lambda_2) > S(\lambda_1)$ and for a given H on the *o.p.* $q(\lambda_2) > q(\lambda_1)$ which implies $h(\lambda_2) > h(\lambda_1)$. Hence, we can conclude:

(e) An increase in λ causes, in a given zone, an increase in the growth rate h at a given level of housing, H, and vice versa if λ decreases.

(f) An increase (decrease) in λ causes the steady state level $H(\infty)$ of a given zone to increase (decrease).

(g) From (16) we see that an increase (decrease) in λ causes the boundary of the city to move away from (nearer to) the CBD.

By the boundary of a city at a given time we mean the first zone where development does not take place at the given time. This zone, as well as those beyond it, can be populated; but they are not being developed at the time in question. In other words, the boundary of a city is the nearest zone to the center at time t which fulfills $h(t) = 0$.

We define the city limit as the nearest zone to the CBD ever to be settled and denote it by $i = LS$.

Proposition (h) follows now from the discussion above.

(h) If $\lambda(t) < \infty$ for $0 \leq t \leq \infty$,

then the city limit is finite—i.e. $LS < \infty$,

$H_i(t) < \infty$ for $0 \leq t \leq \infty$ and for all $i < LS$

and

$$n(t) < \infty \text{ for } 0 \leq t \leq \infty.$$

In Fig. 4 the three main types of growth are described. The subareas are described continuously according to their distance X from the center. H, in this case, denotes the density. In Fig. 4a the case of constant λ is

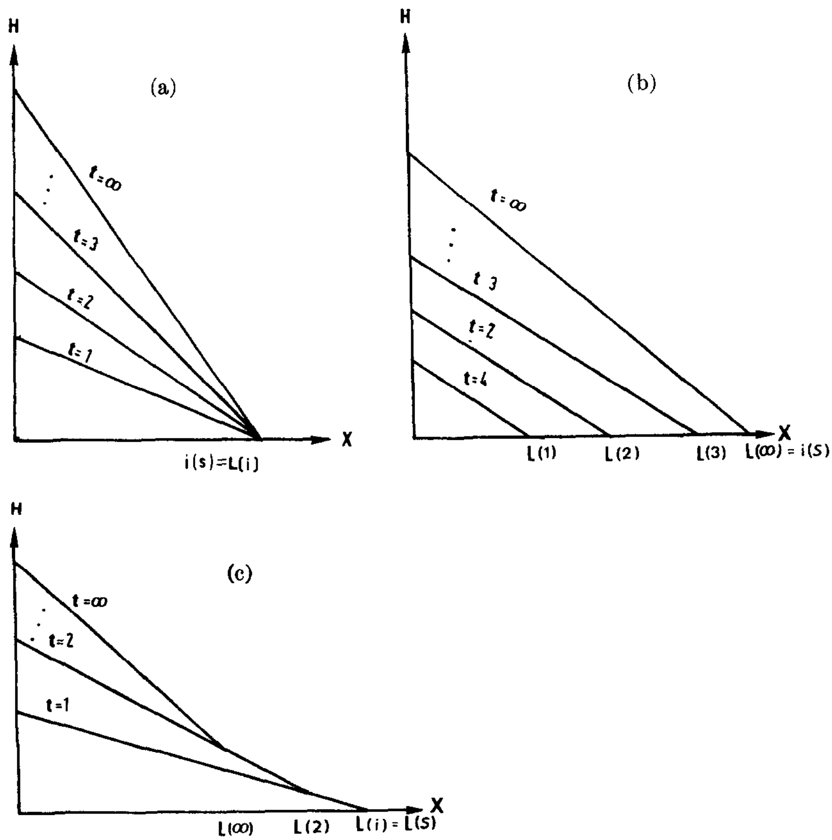

FIGURE 4

described, in Fig. 4b the case of $\lambda > 0$, and in Fig. 4c the case of $\lambda < 0$. Note that at a given time t, the population dispersion described is the familiar one, known from the static analysis models. Note too that a given initial condition $H_i(0)$ and given converging schedule $\lambda(t)$ corresponds to one and only one converging schedule, $n(t)$, and vice versa. *Hence the above discussion is a description of the solutions of equations (7) to (10) for different converging schedules of $n(t)$ for the case where $g_{hh} > 0$ for all zones.*

However, the growth pattern is more likely to be a combination of two or even all three growth patterns described above. At some time, the city boundary expands quickly (i.e., $\dot{n}$, $\lambda \gg 0$) and at other times the city boundary expands little or not at all (i.e., $\lambda \geqq 0$). λ might even become negative and the boundary of construction activity would retreat. Then the boundaries might advance again rapidly and so on. The more rapid the development rate, i.e., the larger $\dot{n}$ and hence λ, the more sprawling and scatteration will happen in the city. If we compare two optimal cities of the same size, one of them young and growing rapidly, the other growing slowly, and hence being older than the first, we will find that the younger city is more scattered than the second. We conclude as follows:

Conclusion 1. The quicker an optimal city grows, the more sprawled it will be. Therefore among cities of equal size the younder city will tend to be more sprawled.

IV. THE SOLUTION FOR THE CASE $g_{hh} < 0$

In Section I we concluded that this case can hold only in densely populated rehabilitation zones. Such conditions can exist in a few zones near the CBD, never in the whole town. In this section, however, we assume as a working condition only that $g_{hh} < 0$ for all levels of h and H. We prove the following lemma:

Lemma. Given a cost function $g(H, h)$ satisfying 4.1 to 4.5 and $g_{hh} < 0$, the solution $[H_i(t), h_i(t), q_i(t), n(t)]$ to the optimization problem of minimizing (5) subject to (1), (2), and (6), will:

(a) Satisfy equations (7) to (10).

(b) For every t, there exists an integer k satisfying $h_k = \dot{n}$ and $h_i = 0$ for $i \neq k$. *Proof:* Equations (7) to (10) are the necessary and sufficient conditions to be met. To prove b, assume there exists $0 < t < \infty$ and at least two indices, i_1 and i_2, which simultaneously fulfill $h_{i_1}(t) > 0$ and $h_{i_2} > 0$ and prove a contradiction which implies the lemma. Without loss of generality assume $i_1 = 1$ and $i_2 = 2$, and $q_1 \leq q_2$, and let $h_i = h_i^0 > 0$ be the alleged optimal solution.

Consider an alternative solution to the problem, i.e., $h_1^* = h_1^0 + h_2^0$ and $h_2^* = 0$. The rest of the variables remain the same. According to a theorem in control theory, (see, for example [1, Ch. 2], or any standard control theory text) the Hamiltonian is optimized by the optimal controls h_i given the optimal auxiliary and state variables, q_i and H_i. Define by $M(h_1, h_2)$ those parts of the Hamiltonian which include h_1 and h_2, then

$$\begin{aligned} -M^0 &= qh_1^0 + q_2h_2^0 + g(H_1, h_1^0) + g(H_2, h_2^0), \qquad (19)\\ -M^* &= q_1(h_1^0 + h_2^0) + g(H_1, h_1^0 + h_2^0) + g(H_2^0, 0). \end{aligned}$$

Hence, if we prove $M^* - M^0 > 0$ we proved a contradiction.

Expand $g(H_1, h_1 + h_2)$ to a Taylor series around h_1, and expand $g(H_2, 0)$ to a Taylor series around h_2. Then:

$$\begin{aligned} g(H_1, h_1 + h_2) &= g(H_1, h_1) + h_2 g_h(H_1, h_1) + R(h_2, h_1, H_1), \qquad (20)\\ g(H_2, 0) &= g(H_2, h_2) + (-h_2)g_h(H_2, h_2) + R(-h_2, h_2, H_2), \end{aligned}$$

$g_{hh} < 0$ implies $R(h_2, h_1, H_1) < 0$ and $R(-h_2, h_2, H_2) < 0$. Use (19) to calculate $M^* - M^0$, then substitute (20) into the RHS of the result, since h_1^0 and h_2^0 are optimal and different from zero by assumption, equality holds in (7) for both zones which implies $q_1 + g_h(H_1, h_1) = q_2 + g_h(H_2, h_2)$. We then get

$$M^* - M = -[R(h_2^0, h_1^0, H_1) + R(-h_2, h_2, H_2)] > 0, \qquad (21)$$

proving the contradiction and the lemma.

The above lemma implies that at any time it is worthwhile to accumulate all the available resources into one zone, and thus take advantage of scale economies. However, since this causes the density in the zone to increase in time and since $g_{hH} > 0$ this implies an increase in marginal construction costs in this zone. Hence, construction activity will move to the next zone. Eventually, all zones will be built; and since $n(\infty) < \infty$ a stationary state must exist in each zone.[5] All this is summarized in conclusion 2.

Conclusion 2. In rehabilitation areas where massive demolition takes place, the optimal construction policy is to concentrate all construction activity in one zone at a time.

[5] At this stage the question of existence of a stable optimal path may arise. To show stability note that for every i, $H_i(t)$ is a function from $+R^1$ into V, and V is a compact subset of $+R^1$. Where: $V = \{x\colon 0 \leq x \leq n(\infty)\}$. $H_i(t)$ is a continuous non-decreasing function and therefore has a single limit point, $H_i(\infty)$, in V. We thus have proved that for every i there exists $H_i(\infty)$, $0 \leq H_i(\infty) \leq n(\infty)$, so that $\lim_{t\to\infty} H_i(t) = H_i(\infty)$ and $H_i(\infty)$ is unique. Q.E.D.

In order to characterize further the optimal trajectory in this case, more specific assumptions about the cost function are needed. We, however, will not pursue the subject further.

V. THE COMPETITIVE MARKET EQUILIBRIUM

We assumed that there is a homogeneous product called a housing unit, and that an individual purchases the same product everywhere. With a housing unit, the individual has to purchase the whole residential bundle, i.e., commuting and housing maintenance. Since he can purchase the same bundle everywhere, an individual is indifferent between locations; and hence locates where the price is the lowest. Hence, the demand price for the residential bundle is equal everywhere at each t. Designate this demand price of the bundle by $\lambda(t)$. At each location the supply price of the residential bundle consists of three components; and since $\lambda(t)$ is the lowest price in the city, (22) must hold for each zone:

$$\lambda(t) \leq p_h{}^i(t) + p_T{}^i(t) + p_H{}^i(t) \quad i = 1, 2, 3, \ldots \tag{22}$$

and equality holds in a zone when construction activity takes place.

$p_h{}^i(t)$—the price a houseowner pays to the housing producer for a unit of housing at time t in zone i.
$p_T{}^i(t)$—total discounted commuting costs an individual expects to pay while living in zone i from time t to infinity.
$p_H{}^i(t)$—total discounted expected maintenance costs of a housing unit in zone i, beginning at period t.

Assume two states of expectations; the first is a state of perfect foresight, i.e., the expectations exactly match the future. In this case, the discounted commuting costs from period t on are

$$p_T{}^i(t) = T_i/r. \tag{23}$$

Current maintenance costs paid by a homeowner in zone i in period t are determined in the market as described in Fig. 5. The demand curve for maintenance at time t in zone i is totally inelastic at a fixed level of housing stock $H_i(t)$. The supply curve, for a given rate of construction $h_i(t)$ is $g_H[x, h_i(t)]$. Hence, equilibrium cost is $g_H[H_i(t), h_i(f)]$. The total discounted maintenance costs for a housing unit in zone i from time t on is therefore

$$p_H{}^i(t) = e^{rt} \int_t^\infty e^{-rt'} g_H[H_i(t'), h_i(t')]dt'. \tag{24}$$

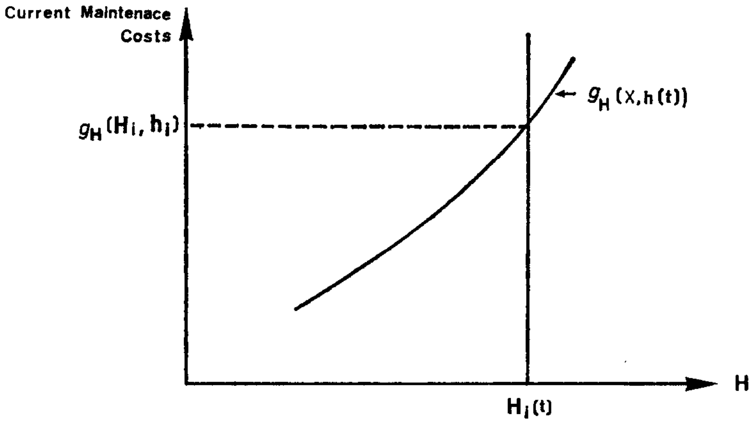

FIGURE 5

Therefore, in the new housing market at each zone the fixed demand price is:

$$p_h{}^i(t) = \lambda(t) - p_T{}^i(t) - p_H{}^i(t) = \lambda(t) - T_i/r - e^{rt}\int_t^\infty e^{-rt'} g_H(H_i, h_i)dt'.$$

The supply curve at time t in zone i is the marginal cost curve $g_h[H_i(t), X]$. When $g_{hh} > 0$ the supply curve rises from left to right. It is shown in Fig. 6 how the equilibrium is derived.

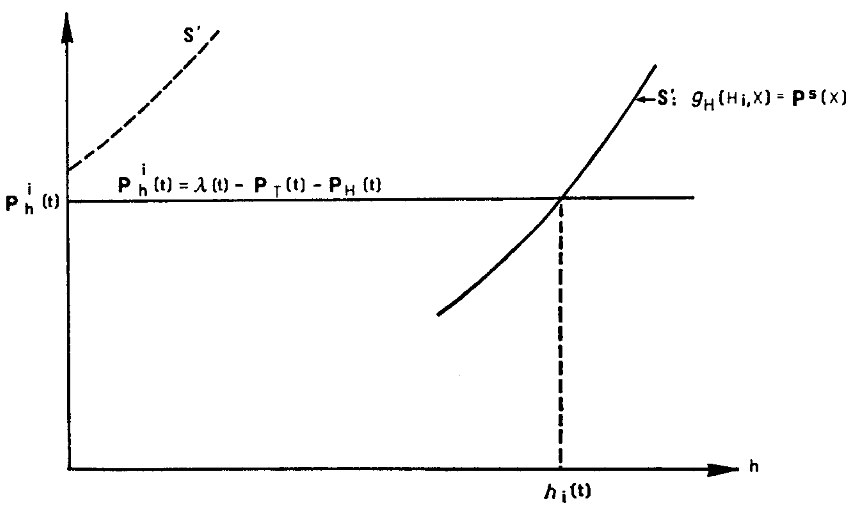

FIGURE 6

In zones where $g_h(h_i, X)$ is completely above $p_h^i(X)$ (see S^1 in Fig. 6), the zone is not built at period t and inequality holds in (22). The full equilibrium equation is, hence, given by (25).

$$\lambda(t) - T_i/r - e^{rt} \int_t^\infty e^{-rt'} g_H(H_i, h_i) dt' - g_h(H_1, h) \leq 0. \qquad (25)$$

and equality holds when $h_i \neq 0$, but (25) is exactly (11). Hence, conclusion 3 follows:

Conclusion 3. Under the assumption of the model with costs of adjustment ($g_{hh} > 0$) and a state of perfect foresight, market equilibrium yields the optimal solution.

If $g_{hh} < 0$, the supply curve in Fig. 6 declines from left to right, hence no stable equilibrium is possible. A possible market solution in this case is a monopolistic solution which might yield the optimal solution.

Now suppose perfect knowledge does not prevail and consumers expect housing maintenance costs to be the same in the future as when the house was purchased, i.e., $g_H[H_i(t), h_i(t)]$. Then the total discounted maintenance costs expected by the consumer are

$$p_H^i(t) = g_H[H_i(t), h_i(t)]/r < e^{rt} \int_t^\infty e^{-rt'} g_H[H_i(t), h_i(t)] dt'.$$

Hence, in this case the demand price for new housing in zone i at time t for a given price $\lambda(t)$ is higher than the demand price in the case of perfect knowledge. This means that for a given population schedule, the city boundary at a given time t is nearer the CBD and each zone is more densely populated than in the optimal case. The opposite is true when expectations are above actual costs.

In recent literature [7, 8, 13, 14], nonoptimality in competitive equilibrium was caused mainly by externalities. Here two more reasons are added: the first is the lack of perfect foresight by people. People living in a city have in mind some rate of development. If the actual rate is greater than what people expect, the city tends to be more compact than the optimal city, and vice versa if expectations are above the actual rate of growth.

In the second case, scale economies in zones where massive demolition is required for further development make competitive market solutions impossible. Conclusion 4 sums up the argument:

Conclusion 4. Cities in competitive equilibrium with very "high" or very "low" rates of development tend to be more dense or more spread out respectively than their respective optimal size.

VI. CONCLUSION AND EVALUATION

To sum up the results let us describe the general picture of a city, as derived from the model. In its early stages, the whole city consists of zones with an abundance of land and other resources. At this stage no costs of adjustment exist and the city grows according to the static way, i.e. the boundary of the city and the density of the population in each zone grows constantly as population increases.[6]

Soon, as zones begin to fill up, costs of adjustment appear. At this stage the growth of the city might acquire a wave-like pattern, i.e. the boundary of construction activity moves at varying rates.

We therefore, expect that cities with a relatively high rate of growth will be more sprawled than cities of the same size but which were built over longer periods of time. The scatteration of Los Angeles compared to older cities can be explained as due to a rapid rate of growth over same period. At a certain stage of the city development, it might reach the point where, in some zones, demolition and reconstruction are needed. It is optimal then to concentrate resources for developing these rehabilitation zones in one zone at a time and perform development in jumps. In such zones no competitive market solution is possible and some other market solution, if any, is needed. Usually government intervention is required in such zones. Sometimes nothing is done and those zones are allowed to deteriorate.

In the optimal case, construction activity takes place either in one of the zones with economies of scale or in all the zones with decreasing returns. While this is not necessarily true for the free market situation, it is nevertheless often the case, since the quantity of housing produced in renewal zones exhausts the total demand for a while. Since the housing constructor in this zone can, if he does produce, outbid his competitors in the other zones.

Return to our basic assumptions and see how they affect the solution. About transportation we assume constant commuting costs over time and hence we do not account for congestion. Introducing congestion into this model would certainly give more results but will not necessarily change the trends discussed above. Actually, the whole transportation activity is exogenous to this model since the only endogenous land use possible in the model is housing. This means that the results of the model apply to the strictly residential areas and that the densities involved here are net densities only. By making the transportation sector endogenous, we will get additional results concerning the transportation

[6] This growth pattern is described by the case $g_{hh} = 0$. For further details contact the authors.

sector and the inter-relations between the two sectors, but the main results achieved here are still valid.

A similar argument applies to the assumption of fixed quantity of housing demand. By introducing a sloped demand function, changing with the real income (net from transportation costs) from zone to zone we are bound to get additional results but the trends identified here should still be valid.

REFERENCES

1. K. J. Arrow and M. Kurz, *Public Investment: The Rate of Return and Optimal Fiscal Policy* (1970).
2. A. J. Frenkel and A. C. Rodriguez, "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach," *American Economic Review* (September 1975).
3. Masahisa Fugita, "Optimal Spatial Patterns of Urban Growth," Regional Science Association, Cambridge, Md. (November 1975).
4. Masahira Fugita, "Spatial Patterns of Urban Growth: Optimum and Market," *Journal of Urban Economics*, **3**, 209–241 (1976).
5. John P. Gould, "Adjustment Costs in the Theory of Investment of the Firm," *Review of Economic Studies*, **35** (1968).
6. Eitan Hochman and Oded Hochman, "On the Relations between Demand Creation and Growth in a Monopolistic Firm," *European Economic Review*, **6**, 17–38 (1975).
7. Oded Hochman, "Market Equilibrium versus Optimum in a Model with Congestion: A Comment," *American Economic Review*, pp. 992–6 (December 1975).
8. Oded Hochman, "A Two Sector Model with Several Externalities and Their Effects on an Urban Setting," *Journal of Urban Economics* (January 1978).
9. Robert E. Lucas, Jr., "Adjustment Costs and the Theory of Supply," *The Journal of Political Economy*, **75**, 321–334 (1967).
10. Edwin S. Mills and David M. deFerranti, "Market Choices and Optimum City Size," *American Economic Review*, **61** (2), 340–345 (1971).
11. Edwin S. Mills, "An Aggregative Model of Resource Allocation in a Metropolitan Area," *American Economic Review*, **57**, 197–210 (1967).
12. Richard F. Muth, *Cities and Housing*, Chs. 2–3, 1969.
13. Y. Oron, D. Pines, and E. Sheshinski, "Optimum versus Equilibrium Land Use Patterns and Congestion Toll," *The Bell Journal of Economics and Management Science*, **4** (2), 619–636 (1973).
14. Y. Oron, D. Pines, and E. Sheshinski, "The Effect of Nuisances and Land Values Associated with Urban Traffic on Suburbanization," *Journal of Urban Economics*, **1**, 382–399 (1974).
15. Michael Rothschild, "On the Cost of Adjustment," *The Quarterly Journal of Economics*, pp. 605–622 (November 1971).
16. Robert M. Solow, "Congestion, Density and the Use of Land in Transporation," *Swedish Journal of Economics*, **74**, 161–173 (1972).
17. A. B. Treadway, "On Rational Entrepreneurial Behavior and the Demand for Investment," *Review of Economic Studies* (1969).
18. Hirofumi Uzawa, "Time Preference and the Penrose Effect in a Two Class Model of Economic Growth," *Journal of Political Economy* (July/August 1968).